The Risk and Return of Impact Investing Funds

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Abstract

We provide the first analysis of the risk exposure and consequent risk-adjusted performance of impact investing funds, private market funds with dual financial and social goals. We introduce a new dataset of impact fund cash flows constructed directly from financial statements. When accounting for market risk exposure, impact funds underperform the market by \$0.30 on the dollar, but outperform venture capital (VC) funds by \$0.15 on the dollar, consistent with the presence of substantial frictions in private markets. Impact funds perform on par with funds matched on size, asset class, and vintage years. We exploit known distortions in measures of VC performance to characterize the risk profile of impact funds. Impact funds have substantially lower market beta than VC funds, contradicting the idea of sustainability as a "luxury good." Adding additional factors does not change the estimate of performance.

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1 Introduction

As major societal problems like climate change and inequality grow, investors and the public have become increasingly interested in whether financial markets can be harnessed to help address these issues. Industry participation in sustainable finance and responsible investing has exploded in recent years, with global sustainable investing assets amounting to $30.7 trillion in 2018, a 34 percent increase in two years (GSIA, 2018).[1] While social and environmental responsibility are often debated in the context of public markets, private markets are uniquely suited to address these challenges because of their dominance in early-stage and growth transactions. At these early stages in companies' lifespans, capital providers exert more influence on both deal sourcing and governance than what they would be able to achieve in public markets (Phalippou, 2020; Gompers et al., 2020).

Impact investing is the practice of using private market strategies to target both financial returns and a social or environmental goal. Although impact investing is a rapidly growing asset classes, with $715 billion in assets under management globally, relatively little is known about the financial properties of this approach (Hand et al., 2020). In particular, no work has addressed the riskiness of impact investing or its financial performance adjusted for market risk exposure, to the best of our knowledge.

This paper fills a gap in the literature by characterizing the risk-adjusted return of impact investing and its risk properties relative to other strategies. Three reasons motivate us. First, establishing risk and return properties is a critical component of understanding the feasibility and future of impact investing—even (perhaps especially) if impact investing is not a strategy that maximizes returns. If impact is concessionary, it is essential to describe the magnitude of the potential risk-adjusted concession in order to understand who will be able or willing to participate in these strategies. Likewise, characterizing the riskiness of impact investing can also illuminate how impact investing fits into existing portfolios.[2]

Second but equally important, a large debate over the returns of impact investing reflects divergent theories of the market. Under standard assumptions (perfect and complete capital markets, rational and informed investors), constrained strategies like impact investing must have lower risk-adjusted returns than unconstrained strategies (Brest et al., 2018; Barber et al., 2021). If these assumptions fail, however, the inequality need not hold. This is in line with recent work by Cole et al. (2020), which finds that markets are imperfectly integrated and that impact investing extracts value from this

[1] Covers five markets: Australia and New Zealand, Canada, Europe, Japan and the United States.

[2] Some have argued that impact investing is only meaningful when it has additionality, and that impact investments should be motivated by a moral imperative rather than a financial one (Phalippou, 2020). We do not disagree with these positions. Our priority is to assess the risk of such an approach.

market friction, leading to potentially higher returns than unconstrained strategies. Risk is a critical part of this debate: if low absolute returns for impact reflect a low covariance with the market, then it is possible that they are profitable on a risk-adjusted basis. Profitable risk-adjusted returns for a constrained strategy would provide evidence in favor of the violation of standard market assumptions. Under incomplete markets, concessionary *absolute* returns may still be consistent with a profitable strategy if the strategy hedges risks.

Third, the risk profile of impact investing also sheds light on different models about the risk of sustainable and green assets. The covariance of impact investing with the market is in theory ambiguous. On one hand, Bansal, Wu, and Yaron (2018) document that socially responsible investing (SRI) in public markets is highly pro-cyclical. Extending this theory to private markets, we might expect the market beta of impact investing to be high, and higher than comparable nonimpact private strategies.[3] On the other hand, Nofsinger and Varma (2014) and Pástor and Vorsatz (2020) find evidence that sustainable mutual funds outperform during market crises. Recent work by Gibson et al. (2019) and Wang and Sargis (2020) suggest that ESG investing in public markets can reduce portfolio risk. This counter-cyclical view of impact investing would correspond to a lower market beta than nonimpact strategies. There are additional risk factors that an impact investor may be interested in hedging. For example, evidence indicates that climate risk is a growing concern for investors (Krueger et al., 2020), and that it may be mispriced in financial markets (Andersson et al., 2016; Engle et al., 2020). By focusing on solutions to climate and other risks, impact investing may serve as a hedge for downside risks, implying a potentially lower beta than comparable nonimpact strategies.

Private market funds in general, and impact investing funds in particular, present several challenges for studying risk and return. Infrequent and highly skewed payoffs make it difficult to use traditional linear factor modeling techniques. Small sample sizes and young funds exacerbate these problems. To address these issues in the VC literature, Korteweg and Nagel (2016) develop the *Generalized Public Market Equivalent* (GPME), an extension of the *Public Market Equivalent* (PME) first proposed by Kaplan and Schoar (2005). As Korteweg and Nagel (2016) show, although the PME has many useful features for assessing the performance of venture capital, it assumes a restricted market equity premium and risk-free rate. As a result, the PME systematically overestimates the performance of high-beta assets in times of rising public equity markets. This distortion grows with the asset's market beta.

Our key insight is to leverage these known distortions to back out the risk properties of impact investing. Given that our sample time period is one of rising public equity markets, the distance be-

[3] Previous work has found evidence of a sizeable market beta for venture capital (VC) investing strategies. For example, Cochrane (2005) finds a beta of 1.9.

tween PME and GPME (the "PME wedge") informs us about the relative market beta of the underlying asset. If impact investing is a high-beta asset, then its PME wedge will be positive; if it is a low-beta asset (beta less than one), the PME wedge will be negative. Moreover, artificially levering up the strategy, and thus amplifying its beta, should amplify the PME wedge: if unlevered beta is positive, the PME wedge will increase. The same logic can be applied to relative properties across strategies. For example, if impact investing has a higher (lower) beta than VC, its PME wedge will be greater (smaller) than the PME wedge of VC. We similarly derive the relative covariance of impact investing with different public factors, such as the sustainability index, by estimating PME and GPME with the alternative factor and comparing the new PME wedge across impact and non-impact strategies. Finally, we extend our model to multiple factors, and test whether impact investing cash flows are spanned by a combination of the market and other factors.

The impact investing cash flow data we use stem from a cross-school collaboration, the Impact Finance Research Consortium (IFRC). As part of the IFRC effort, we collect annual and quarterly financial statements directly from impact funds and manually convert them into a standardized database covering contributions and distributions net of fees, as well as net asset value (NAV), among other variables. This database will be made available for researchers and is an ongoing effort. As of this writing, our dataset contains cash flows for 62 funds from 1999 through 2017. After restricting analysis to funds targeting market-rate returns and with sufficient data, our current sample covers 51 distinct funds.

We construct comparison fund groups using fund cash flows from Preqin.[4] Benchmark comparison groups serve two purposes. First, benchmarking to a well-understood asset class helps us understand performance in an established context. For this purpose, our first comparison group is the universe of US-based VC funds over the same time period as our sample. We choose VC because it is overall the most representative of impact funds (Geczy et al., 2020), and its risk and return properties have been studied extensively (Korteweg, 2019). The second purpose of benchmarking is to try to isolate the influence of the impact component. As a second comparison group, we match each impact fund in our sample to a Preqin fund with the same vintage, asset type, and as close as possible on size. Perfectly controlling for everything outside impact is not possible, but this smaller benchmark helps us to address the effect of characteristics that influence risk exposure.

We find that after properly accounting for risk exposure and the rising market equity premium, impact underperforms the public market by \$0.30 per \$1 invested over the 1999-2017 time period. Market risk exposure alone cannot explain the low returns of impact investing. At the same time, we

[4] In future iterations, we intend to include Burgiss comparisons as well.

find VC underperforms public benchmarks by an average \$0.44 per \$1 invested over the same time period. A portfolio that goes long \$1 in VC and short \$1 in impact has a negative market risk-adjusted return of −\$0.15. Matched funds perform somewhat better, with a loss of "only" \$0.21 per \$1 invested. A portfolio long in matched funds and short impact yields a return of \$0.08, not statistically different from zero.

Our findings provide a new perspective for the debate on financial performance and market completeness. On one hand, we confirm that the constrained impact strategy is concessionary relative to the risk-return frontier that can be achieved in public markets. On the other hand, impact returns are not concessionary relative to VC after accounting for systematic risk – in fact, we find the opposite holds. Since VC could in principle invest in the same deals as impact funds, this points to the failure of one or more assumptions in private markets. Cole et al. (2020) argue that imperfect integration of international capital markets enable profitable impact investing strategies. Other possibilities include information barriers, investor biases, and distortions to competition in both capital and product markets.

Our second main result is that impact investing is substantially less sensitive to movements in public equity markets than VC, and to a lesser extent, than matched funds. The PME overestimates impact performance relative to the GPME in our time period, consistent with a market beta greater than one.[5] At the same time, when we artificially lever up our impact fund cash flows, the PME wedge does not increase as fast as we would expect, and we cannot reject a beta equal to or even below one. What we can reject is that impact beta is close to VC beta. A portfolio that goes long \$1 in VC and short \$1 in impact still has a positive beta, indicating greater market risk exposure for VC than for impact. The difference with matched funds is lower, but still consistent with a particularly low beta for impact. Overall, impact is not as pro-cyclical as we would expect for a "luxury good", or at least not as much as VC. Instead, adding impact investments to a private market investor's VC portfolio reduces overall market risk exposure.

Incorporating other factors like sustainability and small growth into two-factor models with the market does not substantially change our GPME estimates. Neither of these multifactor models span impact, matched, or VC cash flows better than the single-factor market risk model, in the sense that the negative magnitude of absolute returns cannot be explained by exposure to these risk factors. We also consider the model of an investor who cares about sustainability, and invests only in a public market sustainability index. In this setting, impact underperforms the sustainability benchmark, while both benchmarks outperform. This suggests that if investors are targeting exposure to a public

[5] This statement holds true in times of rising market equity premia, which we show is the case for our sample period.

sustainability factor, then impact investing is not a profitable means to gain this exposure. Public and private sustainability factors are distinct.

Our work contributes to the broader literature on SRI and environmental, social, and governance (ESG) factors. Most of this work has focused on public markets, exploring investor taste for "green" strategies (Hartzmark and Sussman, 2019; Krueger et al., 2020; Pastor et al., 2019; Fama and French, 2007) and the pricing of ESG factors (Ilhan et al., 2021; Bansal et al., 2018). Renneboog, Ter Horst, and Zhang (2008) provide an overview of SRI in mutual funds. We extend this work to private markets, which present a different set of challenges and opportunities for sustainability-minded investors. Companies seeking capital in the impact investing market are typically orders of magnitude smaller than public companies, with completely different expected growth paths and risk considerations. The potential for impact is also distinct from public market investments. Investors, via funds, have substantially more influence over the development of portfolio companies than they would in public companies.

Data limitations have kept the literature on impact investing sparse, but this paper joins a few others in the burgeoning literature on impact investing. Geczy et al. (2020) describe contracting practices in impact investing funds, and Green and Roth (2020) develop a theory for the existence of profit-seeking impact investing. Barber, Morse, and Yasuda (2021) provide the first estimates of financial performance of impact funds, using a willingness-to-pay model to show that investors accept 2.5%-3.7% lower internal rates of return (IRRs) for impact funds. In contrast, Cole, Melecky, Mölders, and Reed (2020) find that a large impact investor's long-run returns *outperformed* the market by 15% over nearly seven decades of investing activity. We provide a new perspective by explicitly addressing the risk exposure of impact cash flows. We also introduce a new data set that will be available to researchers to add to the work in this space. An advantage of our data set is that we can differentiate impact funds that are explicitly concessionary from funds that state their intention to achieve market-rate returns. Our current results pertain to the latter only.

Last, this paper contributes to a rich asset pricing literature on the risk and return of PE and VC cash flows, starting with Cochrane (2005) and Korteweg and Sorensen (2010). More recently, Ang, Chen, Goetzmann, and Phalippou (2018) estimate a PE-specific return series using a Bayesian approach and assuming a linear factor structure. Gupta and Van Nieuwerburgh (2019) propose a new method to risk-adjust PE returns and estimate factor risk exposures for each cash flow horizon. We build heavily on the GPME approach introduced by Korteweg and Nagel (2016), leveraging the distortions that they document to back out the risk properties of impact and VC funds. This approach allows us to directly compare VC and impact funds in an intuitive way. A key difference between this method annd Gupta and Van Nieuwerburgh (2019) is that the latter uses expectations of stochastic discount factors (SDFs) from dividend strip prices, while we rely on the realized SDF.

The remainder of the paper proceeds as follows. We describe our data in Section 2 and our approach and predictions in Section 3. In Section 4, we show results relative to market exposure, and in Section 5 we introduce other factors. Section 6 concludes.

2 Data

This section explains our data sources and construction.

2.1 Impact Investing Funds

One of the challenges in studying impact investing is the lack of data. This paper describes a new data source, the Impact Finance Database (IFD), that we developed as part of the IFRC, a collaboration across the Wharton School of Business, Harvard Business School, and the University of Chicago's Booth School of Business. The new dataset will be made available for outside research.[6]

The IFD encompasses multiple datasets, covering four key aspects of impact funds: their financial cash flows, impact reports, legal documents, and management practices. This paper focuses specifically on the financial component of the IFD. With the support of the IFRC, we collect financial statements directly from impact investing funds. These include both audited annual financial statements and intermediate quarterly statements, when available. We carefully convert these raw financial statements into two standardized cash flow panels, one at an annual frequency and the other at a quarterly frequency. The variables that we track include contributions to the fund, distributions out of the fund, and net asset value (NAV). At the time of writing the IFD covers annual cash flows for 62 funds and quarterly cash flows for 48 of these funds. The sample covers vintage years 1999 through 2016, with cash flow data through the end of 2017.

Impact funds include funds that target market-rate returns, as well as funds that target a positive but below-market return. For this paper, we focus on market-rate-seeking funds. We also limit our analysis to funds open for at least three years, and with no data gaps at the annual level. This results in a dataset of 51 funds. We use quarterly data if available (44 funds), otherwise annual data (7 funds). Cash flows reflect returns to LPs net of fees, except for the final distribution when funds are still open at the end of our sample, where we follow Harris et al. (2014) and Korteweg and Nagel (2016) and use NAV as the final distribution. This final cash flow does not account for future fees.

Impact funds share salient characteristics with VC funds. Their function is to raise capital to invest

[6] More information on the data and initiative can be found at https://impactfinanceresearchconsortium.org/.

in private companies, and the legal and compensation structure of impact funds is generally similar to that of VC funds (Geczy et al., 2020). One difference is that impact funds tend to invest across lifecycle stages, with portfolio companies ranging from early stage to mature even within a fund. Impact funds can use both debt and equity, but generally tend to favor equity, which makes VC a better comparison than PE, especially when thinking about how the high leverage structure of PE funds can affect their market beta. Like VC funds, impact funds also tend to hold minority stakes. Common exits for impact fund portfolio companies are sales and redemptions.

2.2 Non-impact Funds

We contrast impact investing funds with two benchmarks: VC funds and a group of private market funds matched on asset class, vintage, and size. Comparing impact to two different benchmark cash flows serves two different purposes. First, VC is a well-understood asset class with a robust market. As described above, VC funds are reasonably similar to impact funds. Examining how the impact market compares to the VC market in a general sense is thus a useful way to understand impact performance in an established context. Second, the purpose of comparing a group of matched funds to impact is to try to isolate the influence of the impact component. This requires matching each impact fund as closely as possible to peer funds on characteristics that may affect risk-return relationships. Our thought experiment is: if an investor has a choice between investing $1 in our benchmark PE markets or $1 in the impact market, what are the implications?

Brown et al. (2015) document strengths and limitations of different commercially available data sets for VC and matched benchmark cash flows. In the current iteration of the paper, we present results using cash flows from Preqin. Preqin gathers information from public sources, direct requests, and requests made under the Freedom of Information Act (FOIA). A strength of Preqin is its accessibility, but a potential weakness is that its sample can be prone to survivorship bias. We intend to repeat the analysis with data from Burgiss. Burgiss provides information management services to institutional investors, and makes anonymized data from these funds accessible to researchers. Burgiss data therefore tends to be less sensitive to survivorship bias, but more difficult to access.

To construct our general VC benchmark, we follow Korteweg and Nagel (2016) and restrict the sample of Preqin funds to funds with at least $5 million in assets, and funds with a US focus. We include venture, early stage, and late stage fund types. To mirror the time period of our impact fund sample, we include VC funds with vintages from 1999 through 2016 and cut off cash flows after 2017. We also drop the funds with less than 3 years of data.

To construct our matched benchmark, we match each impact fund to a fund in Preqin based on general asset class and vintage with the closest size. Because Preqin asset classes are slightly different than the asset classes in the impact sample, we make the following adjustments: we match generalist equity impact funds to balanced Preqin funds and impact generalist, debt, and real estate funds to Preqin general venture funds. This gives us a narrower comparison, but is noisier as a group. We allow for funds with a non-US fund focus, but find that most of the matched sample has a focus on the US. We cut off cash flows for each matched fund based on the final cash flow date in the impact sample. We convert the cash flows to quarterly frequency to better match the structure of the impact fund data.

2.3 Sample Construction and Statistics

Table 1 provides summary statistics for our sample impact and benchmark funds. Impact funds are smaller than the average VC fund, with an average size of $95 million compared to $382 million for VC funds. They are also smaller than the matched group, which has an average size of $154 million. Both absolute performance measures (IRR and multiple) and the PME ratio suggest impact underperforms relative to the benchmarks. We construct PME following Kaplan and Schoar (2005) (see Section 3.1 for more detail). On average, all of the funds underperform the market (i.e., they have a PME less than one). Impact as an asset class is the worst performing group of funds, with an average PME of .77. VC funds have a PME of .88 and matched benchmark funds have a PME of .95.

We have fewer earlier vintage years for our impact funds than for VC funds, and correspondingly fewer cash flows. The matched benchmark funds are explicitly matched on vintage year, and have a similar number of cash flows. Additionally, we plot the histogram of asset classes for the impact fund sample in Figure 1. A majority of impact funds are equity funds from VC or other generalist sub-classes. Debt, real asset, and buyout funds are also represented in the sample, but in much smaller numbers than VC and equity funds.

Figure 2 reports on the timing of distributed cash flows as a percent of fund size for impact funds and VC funds. Overall, the profiles are similar, though statistics are noisier for impact funds at the quarterly level. We plot the smoothed cash flow profile at an annual level in Figure 3. The profiles are still similar, with distributions as a percent of fund size increasing in years 5 through 7 of funds. However, the median impact fund has larger payouts than the median VC fund, in addition to a very large final period NAV distribution payout.

2.4 Public Market Replicating Portfolios

To compute the GPME, we require public benchmark portfolios that replicate the capital accumulation and payouts of our private market funds. We use the 1-month T-bill for the risk free rate and the CRSP value-weighted index for the market return. For additional factors, we use the small-growth portfolio return among the six portfolios underlying Fama and French (1993) factors and the Dow-Jones Sustainability World Index. The T-bill rate, CRSP value-weighted index and small growth portfolio return are from Ken French's website and the Dow Jones Sustainability index is extracted from Bloomberg terminal.

3 Characterizing Risk and Return in Private Market Capital

In this section, we review the predominant measurements of performance in the private equity literature and how their various assumptions distort performance evaluation. We use these distortions to characterize how PME and GPME will behave under different risk properties.

3.1 Public Market Equivalent

Originally developed by Kaplan and Schoar (2005), the PME provides a measure of economic performance for illiquid private equity investments by benchmarking them to what investors would have made, had they invested the same cash flows in the public market. Formally, the PME is calculated as follows:

$$PME = \frac{\sum_t \frac{distribution_t}{1+R_{mt}}}{\sum_t \frac{call_t}{1+R_{mt}}} \tag{1}$$

where *distribution* is a cash flow from the fund back to the investor, *call* is a cash flow from the investor into the fund, and R_{mt} is the total return on the market from the inception of the fund to the time of the distribution or call. Conceptually, each cash flow is discounted by the opportunity cost for a representative PE investor of an equivalent cash flow invested over the same time period in the public market. The PME improves on previous standards of performance (such the IRR and multiple) by accounting for the opportunity cost of capital.

Sorensen and Jagannathan (2015) demonstrate that the PME is also a valid measure of performance from an asset pricing perspective. Given an investor with log-utility preferences, the PME discount rate from time t to $t+1$ can be represented as a stochastic discount factor of the following form:

$$M_{t+1} = \exp(-\log R^m_{t+1}) \tag{2}$$

where R^m_{t+1} is the gross return on the market portfolio. When the cash flows of a PE fund are discounted using this SDF, the PME has the interpretation of an excess return measure accounting for the risk of the market. Importantly, the market return is a proxy for the return on the investor's overall wealth portfolio.

As noted in Korteweg and Nagel (2016), the PME has many useful features for assessing PE fund performance. It is well-suited for the analysis of irregular and skewed cash flows. It also does not require strong distributional assumptions about the return generating process (in papers such as Cochrane (2005) and Ang, Chen, Goetzmann, and Phalippou (2018)).

However, there are important drawbacks to the PME as a measure of performance. Kaplan and Schoar (2005) originally noted that the PME implicitly assumes that systematic risk β is equal to 1. The authors explain that this assumption is the result of the difficulty of estimating risk and return relationships without liquid market prices. Korteweg and Nagel (2016) argue that this assumption is particularly distortionary when equity markets are rising. They use the example of jointly log-normal returns. The SDF of the PME implies the following risk-return relationship:

$$\log E[R_{t+1}] = \log E[R_{m,t+1}] - \sigma^2_m + \beta\sigma^2_m \tag{3}$$

This SDF thus restricts the equity premium to the variance of the market return σ^2_m and the (log) risk-free rate to $\log E[R_{m,t+1}] - \sigma^2_m$. Korteweg and Nagel (2016)'s observation is that this relationship is inconsequential for assets with $\beta = 1$ (leading to the risk-return relationship $\log E[R_{t+1}] = \log E[R_{m,t+1}]$). During times of strongly rising equity markets (when $\log E[R_{m,t+1}] - \log R_f > \sigma^2_m$), the SDF will not accurately adjust for market risk exposure if an asset's β is different from one.

3.2 Generalized Public Market Equivalent (GPME)

Korteweg and Nagel (2016) correct for the PME's risk exposure distortion with a new performance measure, the GPME. The GPME is conceptually similar to the PME, but the single-period SDF used to discount PE fund cash flows takes the following form:

$$M^*_{t+1} = \exp(a - br^m_{t+1}) \tag{4}$$

Under this flexible SDF, the PME is a special case when $a = 0$ and $b = 1$. The authors then compound the single-period SDF in order to find the multi-period SDF that can price cash flows over varying time horizons. In the log-normality example from the previous subsection, this SDF implies the log-linear β pricing relationship in Equation 5 that appropriately accounts for market risk exposure (when a and b are estimated to reflect the market return and risk-free rate).

$$\log E[R_{t+1}] = r_f + \beta(\log E[R_{m,t+1}] - r_f) \tag{5}$$

The GPME takes a benchmarking perspective, asking how an investment adds value to an investor's portfolio that would not be attainable from other factors. It is defined as the sum of discounted distributions minus the sum of discounted contributions, using the multiperiod discount rate:

$$GPME_i = \sum_{j=1}^{J} M_{t+h(j)}^{h(j)} \text{Distributions}_{i,t+h(j)} - \sum_{j=1}^{J} M_{t+h(j)}^{h(j)} \text{Calls}_{i,t+h(j)} \tag{6}$$

For each cash flow j, the SDF is calculated from the first cash flow t to the payoff horizon of cash flow j, represented as $h(j)$. This multiperiod discount rate is constructed by compounding one period exponential discount rates together from time t to time $h(j)$ for each cash flow. Therefore, each cash flow is discounted to time t, including the initial investment. Because of this, the GPME can be described as the NPV of the fund. Under the null hypothesis of no abnormal performance, $E[GPME_i] = 0$.

3.3 Prediction Development

In this section, we explain how we can characterize risk and performance properties of different asset classes using the properties of PME and GPME. We start with predictions regarding market risk, then move to covariance with other factors.

3.3.1 Market Risk Predictions

In periods of rising equity markets (i.e., states of the world when the equity premium is higher than what is assumed under the PME), the PME systematically overestimates the performance of high-beta assets when compared to the GPME, and underestimates the performance of low-beta assets. We use this property to back out the asset's covariance with the market, β.

Assuming jointly log-normal VC and market returns, the difference between the log expected return of the GPME and PME can be represented as:

$$(\beta - 1)(\log E[R_{m,t+1}] - r_f - \sigma_m^2) \tag{7}$$

Notice that this difference reduces to zero when β is one. When β is different than one, our predictions depend on the magnitude of the equity premium.

We say the market equity premium is *sufficiently high* if

$$(\log E[R_{m,t+1}] - r_f - \sigma_m^2) > 0 \tag{8}$$

where $\log E[R_{m,t+1}]$ is the log expected gross return of the market portfolio and σ_m^2 is the variance of the log gross returns of the market portfolio. While this is derived under the assumption of jointly log-normal returns, it should be approximately true in the data. This definition characterizes how much the equity premium observed ex post deviates from what is assumed by the log-utility PME model. If β is greater than 1, then the PME understates the market equity premium and overestimates the abnormal return. The GPME corrects for this distortion. Thus, when the PME overstates abnormal performance ($PME > GPME$) in a one-factor model, we can conclude that β is greater than one.

Our first prediction formalizes the behavior of the SDF when the equity premium is sufficiently high.

Prediction 1 (P1): *When the equity premium is sufficiently high, the parameters of the SDF will not be those of the parameters of the log-utility SDF. That is, for an SDF of the form* $M^*_{t+1} = \exp(a - br^m_{t+1})$, $a \neq 0$ *and* $b \neq 1$.

When Prediction 1 is true, then the distortions of the PME will be relevant for assets with a β different from one. From Equation 7, we derive the following prediction.

Prediction 2 (P2): *When the market equity premium is sufficiently high, PME overstates abnormal performance for high-beta (β>1) assets and understates abnormal performance for low-beta (β<1) assets. If* $PME = GPME$, *then* $\beta = 1$.

We use our predictions to compare the β of impact to the β of a set of benchmark funds. From Equation 7, within the same time period, the distortion of PME relative to GPME depends on the magnitude of β. Defining the PME wedge as $PME - GPME$, we make the following prediction.

Prediction 3 (P3): *When the market equity premium is sufficiently high, and all else equal, the relative magnitude of the PME wedge reflects the relative magnitude of the asset's beta. If* $(PME - GPME)_{Benchmark} > (PME - GPME)_{Imp}$, *then* $\beta_{Benchmark} > \beta_{Imp}$.

We also use artificially levered cash flows to give more accurate bounds on β. This exercise was first developed in Korteweg and Nagel (2016). We simulate increasing β by increasing a leverage factor k applied to fund cash flows and then estimating the PME and GPME of the levered cash flows using the original SDFs. The levered cash flows are calculated as:

$$L_{i,t+h(j)} = C_{i,t+h(j)} + k(C_{i,t+h(j)} - C_{if,t+h(j)}) \tag{9}$$

where $C_{i,t+h(j)}$ are the cash flows from the original fund and $C_{if,t+h(j)}$ are the cash flows from the risk-free rate replicating portfolio which matches the capital call schedule of the original fund while investing in the risk-free asset. We assume $k \geq -1$, i.e. no net short-selling of the original fund cash flows.

When the asset has a positive β with no leverage, imposing more leverage to the cash flows will inflate the β of levered cash flows. Reversely, if the β of the asset is negative with zero leverage, levering up will decrease β. Thus if the unlevered β of the cash flows is positive, then the PME wedge increases as the artificial leverage factor increases. This reflects the fact that the PME becomes a worse measure of performance as β moves away from one.

Prediction 4 (P4): *When the market equity premium is sufficiently high, the PME wedge increases with k if $\beta_{unlevered} > 0$. The wedge decreases with k if $\beta_{unlevered} < 0$.*

Building on Prediction 2, the PME wedge should be positive when k is such that the levered β is greater than one, and negative when k is such that the levered β is less than one.

Prediction 5 (P5): *When the market equity premium is sufficiently high, the PME wedge is positive when k is such that $\beta_k > 1$, and negative when k is such that $\beta_k < 1$. The wedge crosses the x axis when k is such that $\beta = 1$.*

Combining P4 and P5, we can back out approximate asset β by looking at the plot of the PME wedge against the leverage factor k. If the wedge is decreasing, $\beta < 0$; if the wedge is increasing but less than 0 at $k = 0$, then $0 < \beta < 1$; and if the wedge is increasing and greater than 0 at $k = 0$, then $\beta > 1$.

3.3.2 Predictions for Covariance with Other Factors

So far we have limited ourselves to a CAPM model where the return on the market portfolio represents the return on the investor's wealth portfolio. We may care about covariance with other factors, for example a public sustainability factor. We can use the same intuition for different public market indexes that may reasonably capture alternative assets that represent the impact investor's wealth portfolio.

We start by generalizing Equation 7 to any public-market factor X.

$$(\beta - 1)(\log E[R_{X,t+1}] - r_f - \sigma_X^2) \tag{10}$$

Our predictions now depend on the magnitude of the equity premium for X. The equity premium of public-market factor X is *sufficiently high* if

$$(\log E[R_{X,t+1}] - r_f - \sigma_X^2) > 0 \tag{11}$$

The equity premium is *low* (not sufficiently high) if

$$(\log E[R_{X,t+1}] - r_f - \sigma_X^2) < 0 \tag{12}$$

When the equity premium is low, the PME *overstates* the public equity premium and *underestimates* the abnormal return when $\beta_X > 1$. In that case, when $PME_X > GPME_X$, we can conclude that $\beta_X < 1$.

Prediction 6 (P6): *If the equity premium is sufficiently high for a public market factor X, then a positive PME wedge implies a $\beta_X > 1$ for that factor.*

If the equity premium is low for a public market factor X, then a negative wedge implies a $\beta_X > 1$ for that factor.

Similarly, we can apply the logic of P4 and P5 and use artificial leveraging in these one-factor models to back out the covariance structure of impact investing returns relative to public market factor X.

Prediction 7 (P7): *If the equity premium is sufficiently high for a public market factor X, then a positive relationship between the PME wedge and the amount of artificial leverage applied to cash flows indicates a $\beta_X > 0$ for that factor.*

If the equity premium is low for a public market factor X, then a negative relationship between the PME wedge and the amount of artificial leverage applied to cash flows indicates a $\beta_X > 0$.

Finally, we can examine the risk profile of impact investing cash flows more generally using multifactor models. In these exercises, we examine what publicly traded factors span impact investing returns. To undertake this analysis, we test whether impact and benchmark cash flows have abnormal performance when discounted with SDFs that account for different risk factors.

Prediction 8 (P8): *If impact investing returns are spanned by public market factors, then the GPME with respect to these factors is zero when cash flows are discounted with a multifactor SDF.*

A significant non-zero GPME with a multifactor SDF indicates that impact investing cannot be replicated with these public market factors.

4 Impact Investing Funds and Market Risk

In this section, we approximate the return on the impact investor's wealth portfolio with the return on the market and examine implications for risk and risk-adjusted performance relative to the market.

4.1 PME and GPME Estimation with Market Factor

As explained in Section 3, we use two SDFs to price impact and PE cash flows: the PME is abnormal performance when the SDF $M_{t+1}^{PME} = \exp(-\log(R_{m,t+1}))$ is used to price cash flows; the GPME is abnormal performance when the SDF $M_{t+1}^{GPME} = \exp(a - b\log(R_{m,t+1}))$ is used to price cash flows.

We begin by estimating the parameters for M_{t+1}^{GPME}. We follow Korteweg and Nagel (2016) and create public market replicating portfolios of the risk-free rate and the gross market return, for each impact, VC, and matched fund in our sample. The purpose of these public market replicating portfolios is to create a cash flow series for an investor that invests in and receives distributions from public assets at roughly the same time intervals as the fund cash flow series we are interested in pricing. We then use GMM to find the parameters of the stochastic discount factor such that the expected sum of discount cash flows across all replicating portfolios is equal to zero. In order to create a consistent measure of performance, we price public market portfolios that replicate impact, VC, and matched funds, and use the one set of parameters throughout our analysis. More details on the estimation method and the relevant assumptions can be found in Appendix A.

The results of our estimation are in Table 2. In the first column, the PME implicitly assumes that the coefficient associated with the risk-free rate (a) is 0 and the slope coefficient on the market return (b_1) is 1. Our estimates in the second column demonstrate that the ex-post SDF is very different from the PME assumptions. Using the procedure described above to estimate the realized SDF, we find the slope coefficient on the log market return is 3.765 and the coefficient associated with the risk-free rate is 0.180. These estimates are consistent with relatively "hot" equity markets and Prediction 1.

We examine the magnitude of the market equity premium more directly by computing a sample estimate for our sample period:

$$\log \bar{R}_m - r_f - \hat{S}_m^2 \approx 0.003 > 0$$

where $\log \bar{R}_m$ is the natural logarithm of sample expected gross return of the market portfolio and $\hat{S}_m^2$ is the sample variance of the log returns. This is consistent with a *sufficiently high* market equity premium during our time period. The market risk of impact funds is thus a relevant concern for understanding performance, and PME will distort performance upwards for riskier assets.

From equation 6, fund-level GPME is given by:

$$GPME_i = \sum_{j=1}^{J} M_{t+h(j)}^{h(j),GPME} C_{i,t+h(j)}$$

For fund i, cash flow time j, cash flow horizon $h(j)$, and cash flows C. Fund-level PME is given as:

$$PME_i = \sum_{j=1}^{J} M_{t+h(j)}^{h(j),PME} C_{i,t+h(j)}$$

In Table 3, we report the average performance of impact and benchmark (VC and matched) funds under PME and under the market risk GPME SDF, using the SDF parameters from Table 2. We report standard errors in parentheses and p-values of the J-test of whether $(G)PME = 0$ in brackets.[7]

Panel (a) presents our results for impact funds. We find a negative impact PME of -\$0.16 per \$1 of capital committed. The p-value is small, but this in part reflects the fact that the SDF parameters for the PME are assumed ($a = 0$ and $b = 1$) rather than estimated. We also find that the market risk GPME of impact funds is negative, but we cannot reject zero pricing errors when adjusting for parameter estimation error. After properly accounting for risk, impact funds underperform the market index by \$0.30 per \$1. The overestimation of the PME relative to the GPME suggests that impact has a market β greater than 1, in line with Prediction 2. We caution that these results are suggestive, due to the relatively small value of the J-statistic for the impact GPME.

However, impact's underperformance relative to the market appears to reflect the broader underperformance of VC over the same time period. In panel (b), we show the VC PME is similar to the impact PME: an abnormal loss of \$0.13 per \$1 of capital committed, and we cannot reject zero pricing errors. The VC GPME is lower than the impact GPME, at a risk-adjusted loss of \$0.44 per \$1 of

[7] For our J-test, we adjust the spectral density matrix for correlation between pricing errors as well as estimation error when applicable (see Appendix A for more details).

capital committed and we can reject zero pricing errors. As with impact, the PME overestimates the performance of VC. These results are consistent with a β of VC larger than 1 (Prediction 2), as has been found in previous literature (see e.g. Boyer et al. (2018),Cochrane (2005)). Appropriately accounting for risk exposure suggests that an investor would be better off adding impact to her portfolio than VC; however, this same investor would experience an overall deterioration in portfolio performance with the addition of either asset class.

The comparison to a group of matched funds tells a different story. We report the results for the matched benchmark funds in panel (c). We find a larger and statistically insignificant PME of -\$0.03 per \$1 capital committed for this group, and an insignificant GPME of -\$0.21. These results suggest the matched group β is greater than one. The wedge is larger than impact, but smaller than VC, suggesting that impact's market risk exposure is less than comparable private equity funds. A public market equity investor would seem to do slightly better investing in the matched group of funds than in impact. We evaluate this claim next.

We directly test the performance of impact relative to the two PE benchmarks by creating portfolios of cash flows that go long \$1 in benchmarks and short \$1 in impact funds. This is not a replicable strategy, but is meant to test relative differences in performance between the benchmarks and impact. The GPME of each long-short strategy tells us the risk-adjusted return to "going long" in the benchmark and "shorting" impact, accounting for equity market performance. We also calculate the PME for each strategy, which assumes a $\beta = 0$ for the set of net cash flows. In this setting, the wedge between the PME and GPME estimates demonstrates the difference in risk profiles between the two sets of cash flows: if the β of impact and the benchmarks are the same, then the PME and GPME of the long-short portfolio should coincide.

We can formalize our analysis in the context of the jointly log-normal model used at the outset of Section 3 in Equation 7. From equation 10, the wedge for benchmark B under jointly log-normal returns is:

$$(\beta_B - 1)(\log E[R_{m,t+1}] - r_f - \sigma_m^2)$$

The wedge for impact is then:

$$(\beta_{Imp} - 1)(\log E[R_{m,t+1}] - r_f - \sigma_m^2)$$

This implies that the wedge of the net cash flows is:

$$(PME_B - PME_{Imp}) - (GPME_B - GPME_{Imp}) = (PME_B - GPME_B) - (PME_{Imp} - GPME_{Imp})$$

Which can then be written as

$$(\beta_B - 1)(\log E[R_{m,t+1} - r_f - \sigma_m^2) - (\beta_{Imp} - 1)(\log E[R_{m,t+1}] - r_f - \sigma_m^2)$$

Given that the equity premium is the same in both samples, this expression reduces to:

$$(\beta_B - \beta_{Imp})(\log E[R_{m,t+1}] - r_f - \sigma_m^2) \tag{13}$$

Given a large market equity premium (relative to σ_m^2), a positive wedge is indicative of $\beta_B - \beta_{Imp} > 0$. A negative wedge indicates that $\beta_B - \beta_{Imp} < 0$. If the equity premium was small (i.e., $\log E[R_{m,t+1}] - r_f < \sigma_m^2$), then the opposite predictions would hold.

We start with the full VC benchmark. For each vintage year in the impact fund sample, we create a long portfolio of VC funds and a short portfolio of impact funds by taking size-weighted averages of all VC or impact funds in the same vintage year. We normalize the contributions such that the sum of contributions to each of these VC portfolios must add up to \$1. This results in 14 long-short portfolios of cash flows, one for each vintage year. We use the SDF estimates from Table 2 and price the cash flows from these portfolios. We create 14 portfolios of matched long-short portfolios in the same way. We also create equal-weighted portfolios and report these results in Appendix B.

Table 4 presents the results. The first column contains the PME estimates that assume the strategy has $\beta = 0$, effectively discounting the long-short cash flows by the risk-free rate. The second column contains the GPME estimates. Panel (a) presents the estimates for the VC-impact long-short strategy and Panel (b) for the matched group-impact long-short strategy. We first test the VC-impact strategy. We cannot reject zero abnormal performance under the assumptions of the PME. Appropriately risk-adjusting the cash flows with the Market Risk SDF from Table 2 leads to a significantly negative return to going long VC and shorting impact, equal to \$.15 per \$1 of capital committed.[8] Thus, we reject that VC outperforms impact over the time period. We can also conclude that the β of this strategy is not zero because the GPME and PME estimates are not the same. In line with Prediction 3, our results indicate that $\beta_{VC} > \beta_{Imp}$.

The returns to going long the matched sample and shorting impact are much different. We find a statistically significant PME of \$.08 per \$1 capital committed and an insignificant GPME of \$.08. The estimates suggest going long in the matched sample is a profitable strategy, but the p-value is large and we cannot reject zero pricing errors. Additionally, the wedge between the PME and GPME in

[8] We treat the parameter estimates as fixed for this exercise; therefore, they do not account for parameter estimation error as in the previous exercise.

these panels is roughly zero, potentially slightly positive. This suggests $\beta_{Matched} - \beta_{Imp} > 0$, although the βs are very close.

We examine the relative β of both strategies in more detail in the next section.

4.2 Backing Out Market Risk Exposure: Artificial Leverage

In this section we use artificial leverage to bound the β on impact relative to our two benchmarks. We use the SDF estimates in Table 2 to price levered cash flows:

$$L_{i,t+h(j)} = C_{i,t+h(j)} + k(C_{i,t+h(j)} - C_{if,t+h(j)}) \tag{14}$$

Where i is fund, t is time, j denotes cash flow time, $h(j)$ denotes time horizon, C_i are fund cash flows, k is a leverage factor, and C_{if} are the cash flows from the fund's public market replicating portfolio for the risk-free rate.

We plot the PME wedge for impact and both benchmarks in Figure 4. We create 95% confidence interval by bootstrapping the wedge estimates 1,000 times. At $k = 0$, there is no additional leverage and the wedge is the difference between the PME and GPME estimates as in the Section 4.1. As we increase k to a leverage factor of 1 and 2, we artificially increase the β of the cash flows. Thus, for any asset with a positive β, we expect an increasing wedge with the addition of more leverage (Prediction 4). If an asset is already a high-beta asset at $k = 0$, this additional leverage should lead to an increasing wedge as the PME becomes a more distortionary measure of performance. If an asset has a $\beta = 1$, then the GPME and PME should coincide at $k = 0$ and we would expect to see a positive slope with the addition of more leverage, crossing the x axis at $k = 0$.

We can also apply negative leverage to the cash flows, replicating a decrease in β. Given a high-beta asset, we can determine how much de-leveraging is necessary to achieve a $\beta = 1$ by examining at what value of k the wedge is equal to zero. As k nears -1, the wedge should become negative as levered β falls below one and the PME begins to understate performance.

The line for VC funds is consistent with Predictions 4 and 5: as the leverage factor and thus β increase, the PME wedge increases as well, as we would expect for a high-beta asset. The wedge is also close to 0 around leverage factor -0.5. These two findings, along with those from the previous section, suggest a VC β of approximately $1/0.5 = 2 > 1$.

The wedge for impact appears relatively constant across leverage factors, only slightly increasing in k. A constant wedge would imply a β close to zero, as additional leverage does not affect the wedge magnitude. However, we can reject a zero wedge at $k = 0$ with 95% confidence, suggesting that β must be at least one. The relatively flat slope of the wedge line is thus surprising. We cannot infer more specifics about the absolute level of impact β from Figure 4, but we can rule out that the sensitivity is as large as VC. This reinforces our conclusion from Table 4 that $\beta_{VC} > \beta_{Imp}$.

The matched fund wedge is flatter than the VC line, but steeper than impact. The 95% confidence intervals are wider for this group of funds than for impact at $k = 0$, so we cannot necessarily reject that β is greater than 1. However, the steepness of the slope relative to the slope of the impact wedge is evidence in favor of the previous section's findings: although the β of the matched group appears slightly higher than impact, it is much closer than the VC benchmark. This finding suggests that impact as a strategy has different market risk exposure beyond risk factors such as size and asset class.

To supplement this analysis, we also apply artificial leverage to the long-short cash flow portfolios from the previous section. Applying artificial leverage to the net cash flows is another way to test both whether the β of the benchmarks and impact are different and whether the β of the benchmarks are greater than the β of impact. If a benchmark and impact β are the same, then the β of that long-short portfolio should be zero, and the PME wedge should stay constant as we increase k.

Instead, what we observe in Figure 5 is a wedge that is increasing in k for both benchmarks. For the VC benchmarks, we can reject a zero wedge at most values of k, which is evidence that the GPME is not equal to the PME and the β of VC is not equal to the β of impact. For the matched benchmark, we cannot reject a zero wedge at any value of k, which suggests that the overall strategy β is close to zero. The positive slope of the VC line indicates that the β of the net cash flows is positive: as k increases, the GPME becomes more negative and the wedge increases. This is further evidence that the β of VC is greater than the β of impact. The slope of the matched benchmark line is much flatter, but still positive. This reflects a slightly larger matched group β that is still very close to the impact β. At $k = 0$, the VC long-short wedge is between 0.17 (value-weighted) and 0.18 (equal-weighted). This is a large difference, and refutes the idea of impact investing as a "luxury good" that is particularly pro-cyclical—or at least, no more than VC is a luxury good. At $k = 0$, the matched benchmark long-short wedge is between $-.003$ (value-weighted) and -0.008 (equal-weighted). The confidence intervals are very wide for these estimates, so we caution putting too much weight into the point estimate of the wedge. However, we see these results as evidence that impact investing provides market hedging services beyond its features (size, asset class) as a private equity class.

5 Impact Investing Funds and Other Factors

Section 4 characterizes the market β of impact funds, i.e., the covariance of these cash flows with the market. In this section, we use the same tools to characterize the covariance of impact fund cash flows with other factors.

5.1 Abnormal Performance and Risk Exposure to Other Factors

We can approximate the covariance of impact fund cash flows with any factor by replicating the analysis in Section 4, using a one-factor SDF with the alternative factor of interest in place of the Market Risk SDF. Predictions 6 and 7 highlight that the interpretation of the analysis depends on the magnitude of the public market premium with that factor. If the market premium for the factor of interest is sufficiently high, then predictions are the same as for the market factor. However, if the market premium is low, our predictions flip.

We investigate impact and VC funds' risk exposure to a sustainability index factor. We examine the magnitude of the equity premium on the sustainability index by computing its sample estimate:

$$\log \bar{R}_{SI} - r_f - \hat{S}_{SI}^2 \approx -0.0007 < 0$$

where $\log \bar{R}_{SI}$ is the natural logarithm of sample expected gross return of the sustainability index and $\hat{S}_{SI}^2$ is the sample variance of the log returns. This corresponds to the scenario in which the equity premium is low—though notably, the absolute value of this premium is an order of magnitude smaller than the market premium, and the distortions should therefore be smaller. We are in the second case of Prediction 6: a negative PME^{SI} wedge would imply a $\beta^{SI} > 1$ with the sustainability index.

Table 5 reports the one-factor SDF estimate and Table 6 gives the PME and GPME estimates using the sustainability index as the public market return of interest. Our estimates show that the ex-post SDF using sustainability as the benchmark is also very different from the SDF with log-utility assumptions. We find a slope coefficient on log sustainability returns of 2.27 and an intercept coefficient of 0.05. The PME and GPME estimates are different between impact, VC, and matched funds. With risk adjustment relative to the sustainability index, VC funds gain \$0.14 dollar and matched funds gain \$0.22 dollar per \$1 capital committed, while impact funds lose \$0.10 dollar per \$1 capital committed. The overperformance of benchmark funds relative to the sustainability index is consistent with a concessionary return for sustainable strategies.

The PME overestimates impact fund performance and underestimates VC and matched fund performance relative to the $GPME^{SI}$. However, since we cannot reject zero pricing error in any of the specifications, we cannot draw conclusions on sustainability β by looking at the "wedge" with zero leverage.

In Figure 6, we show what happens to the wedge for different levels of artificial leverage. Since the sustainability premium is relatively small, we are in the second case of Prediction 7: an increasing wedge, as we see for VC funds, implies negative covariance with the sustainability index. A decreasing wedge, as we see for impact and matched funds, implies positive covariance with the sustainability index. Interestingly, the wedge for matched funds appears to decline faster with leverage than the wedge for impact funds, which would suggest a higher positive covariance with the public sustainability index for matched funds relative to impact funds. The bounds on these estimates are large and the results should be taken with caution, but they highlight that sustainability in private markets, for example via impact investing, is distinct from sustainability in public markets.

5.2 PME and GPME estimation with two factors

The one-factor GPME model can be extended to incorporate multiple factors. The SDF in this case is

$$M_{t+1}^{GPME} = \exp(a - b' \log(F_{t+1}))$$

where f is the number of public market factors, b is an $f \times 1$ vector of factor loadings, and $\log(F_{t+1})$ is $f \times 1$ vector of public market factor returns at time $t+1$.

We use the simplest multi-factor GPME with $f = 2$ to test whether impact, VC, and matched fund returns can be spanned by additional risk factors. We augment the Market Risk SDF with a sustainability factor or the small-growth portfolio from Fama and French (1993). The results are in Table 7. As in Table 3, Panel (a) provides estimates for impact funds, Panel (b) for VC funds, and Panel (c) for matched funds. In the first column, we repeat the GPME estimates from the one-factor market risk model as a reference. The last two columns correspond to two-factor SDFs with the market and small-growth portfolio (second column) or with the market and sustainability index (third column).

Our first observation is that, consistent with Korteweg and Nagel (2016), adding the small-growth factor has little effect on the GPME for VC funds. We get similar results by adding the sustainability index instead of the small-growth factor. For VC funds, we find a negative GPME of −\$0.41 to −\$0.44 per \$1 of capital committed in all three specifications.

We run the same three models on our impact funds and also find close results across specifications.

Similar to Market Risk SDF, we get an abnormal performance of $-\$0.30$ per \$1 invested after adding the small-growth factor. Adding the sustainability index yields an estimate of $-\$0.31$.

For matched funds, we find an abnormal performance of $-\$0.20$ per \$1 committed after adding the small-growth factor, which is similar to $-\$0.21$ with the Market Risk SDF. Adding the sustainability index to the market factor generates a slightly more negative performance, with $-\$0.29$ per \$1 committed, but the standard errors are wide and the difference is not statistically significant.

For VC funds, we can reject zero pricing errors in all models, and find a significant abnormal loss both economically and statistically. We cannot reject zero pricing errors for impact and matched funds due to small sample size, though note fairly low estimates for these funds as well. Whatever additional factor we add, impact funds perform better than VC funds and worse than matched funds after appropriate risk adjustments, consistent with Section 4. Neither of the multifactor models span returns better than the single-factor market risk model. The negative abnormal returns we get in the market risk model cannot be explained by the relative performance of public sustainability-favoring stocks or the small-growth stocks.

6 Conclusion

In this paper, we provide a characterization of the risk profile and risk-adjusted performance of impact investing. To do this, we develop a new approach to derive risk properties of private market asset classes, building on insights from Korteweg and Nagel (2016). While we apply it to impact investing in this paper, our approach can easily be extended to examining other private market strategies in the future. We use this to show, for example, that the market beta of impact funds is statistically significantly lower than the market beta of VC funds. When accounting for market risk exposure, impact funds underperform the market by \$0.30 on the dollar but outperform VC funds by \$0.15 on the dollar. They perform more closely to a set of private funds matched on vintage year, asset type, and size.

Our findings shed light on theories of the market as well as the nature of green assets. Our finding that impact underperforms both the S&P 500 and a public sustainability index is consistent with impact investing as a constrained strategy that necessarily leads to lower returns. We find a similar effect for VC and matched benchmark funds, suggesting that private markets violate perfect and complete market assumptions. Our finding that impact outperforms VC funds is consistent with considerable market frictions in private markets overall. Impact investors may be constrained, but seem to be able to capture value that general VC investors miss, though not matched fund investors.

This could be due to information barriers, investor biases, or distortions to competition in both capital and product markets.

Risk is an important element in this story. Our finding that the beta of impact is lower than the beta of benchmarks, and in particular VC, is consistent with a counter-cyclical interpretation of impact strategies. Although absolute measures of performance are lower during periods of hot equity markets, impact investing acts as a relative hedge against downside risk.

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Figure 1: Distribution of Asset Classes of Impact Sample

We plot the percentage of impact funds in each asset class. VC funds are equity funds that invest with an early stage focus. Other equity funds include late stage and more generalist funds. Buyout funds are equity funds with a buyout focus that use leverage. Debt funds are private funds that originate loans to portfolio companies. Real asset funds invest in physical assets. The remainder of impact funds are generalist, that invest with a variety of styles in companies at various stages.

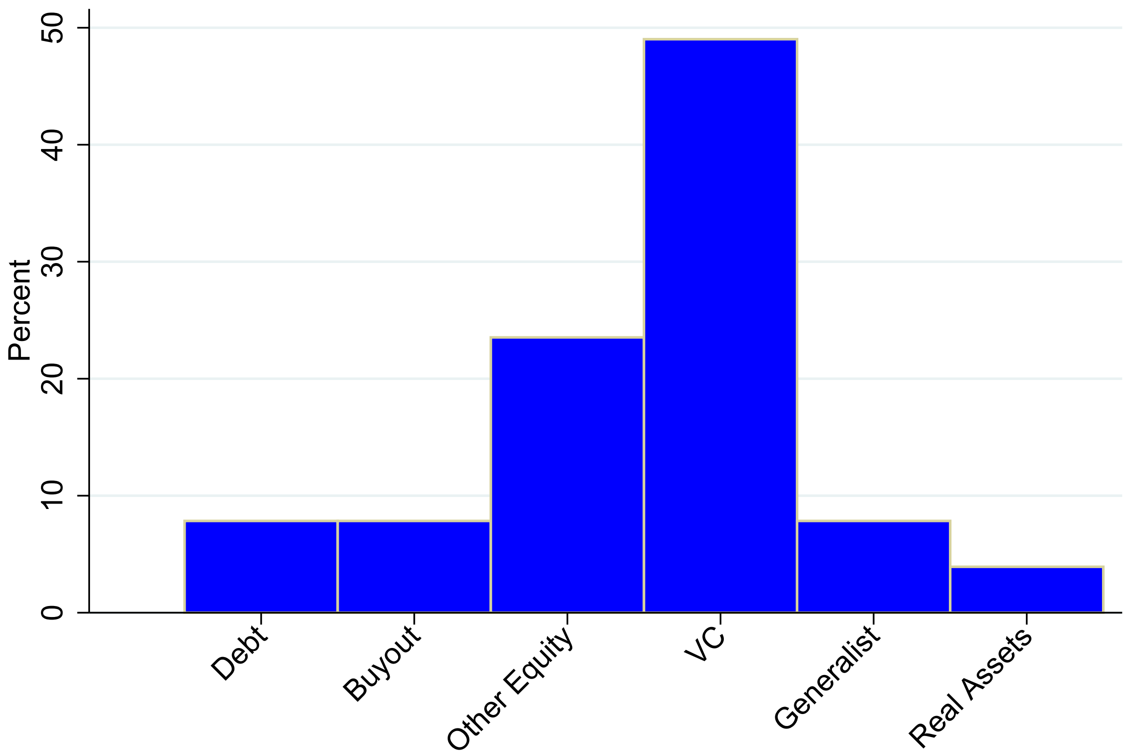
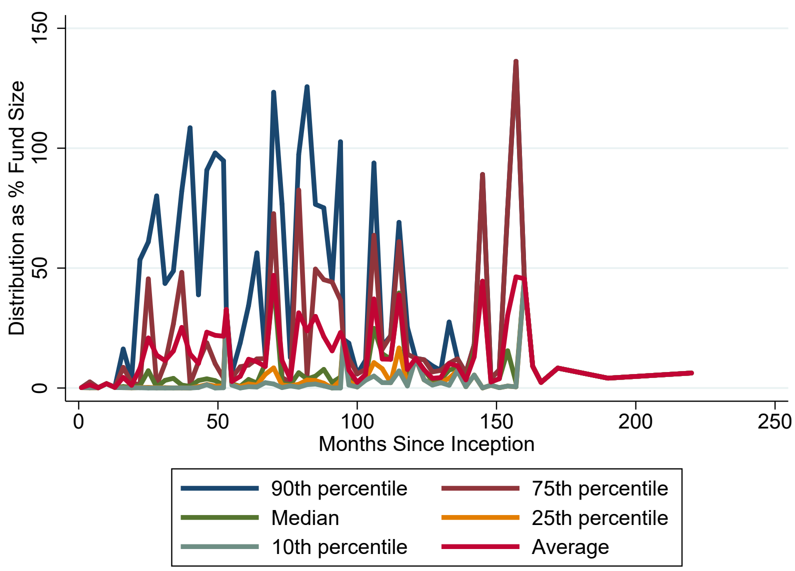


Figure 2: Distribution as % of Fund Size by Month Since Inception

We plot percentiles of distributions normalized by fund size in each quarter since fund inception. We take percentiles across each quarter that a fund is open in order to characterize the lifespan of the fund. This characterizes the cross-section of fund cash flows at each quarter of fund life. The top panel considers the cross-section of impact funds and the bottom panel plots the percentiles for VC funds. We plot the 10th, 25th, 50th, 75th, and 90th percentiles in addition to the average. The timing of the distribution profile looks similar, although the impact fund sample is considerably noisier than the VC sample.

(a) Impact Funds



(b) VC Funds

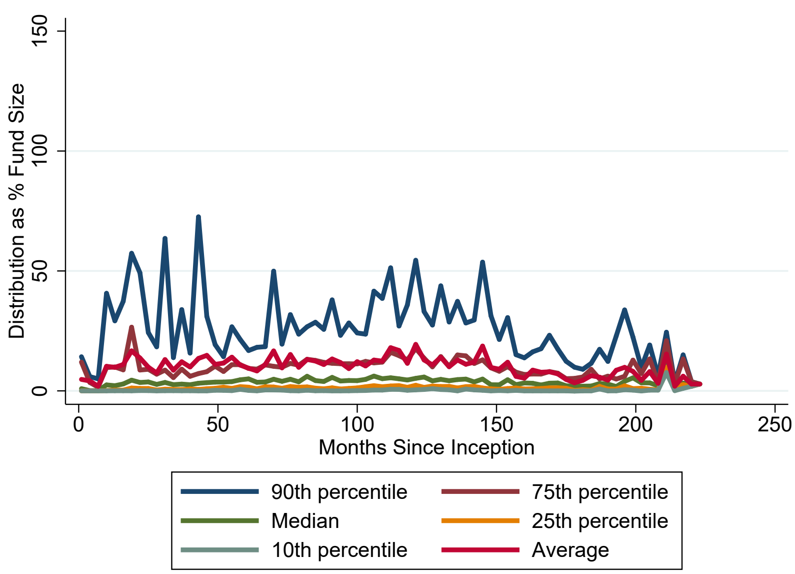
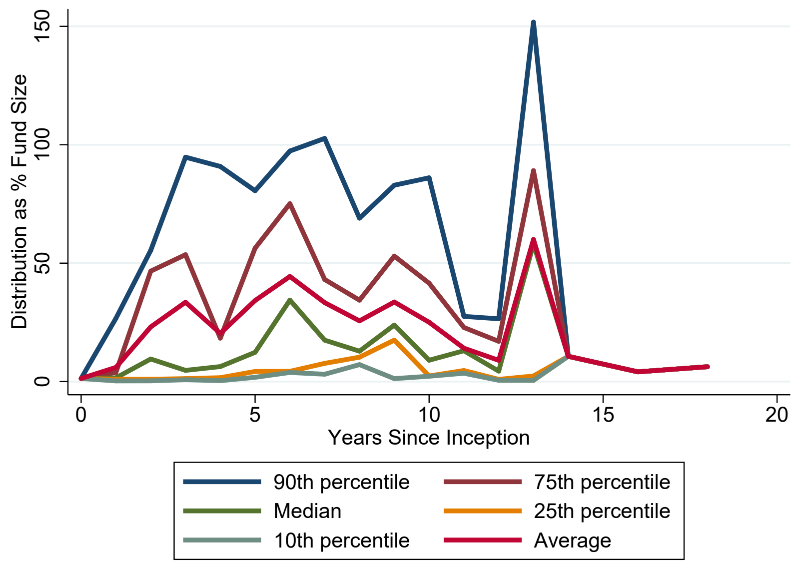


Figure 3: Distribution as % of Fund Size by Year Since Inception

We plot percentiles of distributions normalized by fund size in each year since fund inception. This results in a smoothed version of Figure 2. We sum total distributions in a given year (including the final period NAV) and divide by the total committed capital of the fund. We take percentiles across each year that a fund is open in order to characterize the cross-section of fund. As in Figure 2, the distribution profile looks similar, with an increase in distributions as a percent of fund size around years 5 to 7. The impact funds have very large final period NAV payouts compared to VC funds. Impact's median distribution at each year is higher than for VC funds.

(a) Impact Funds



(b) VC Funds

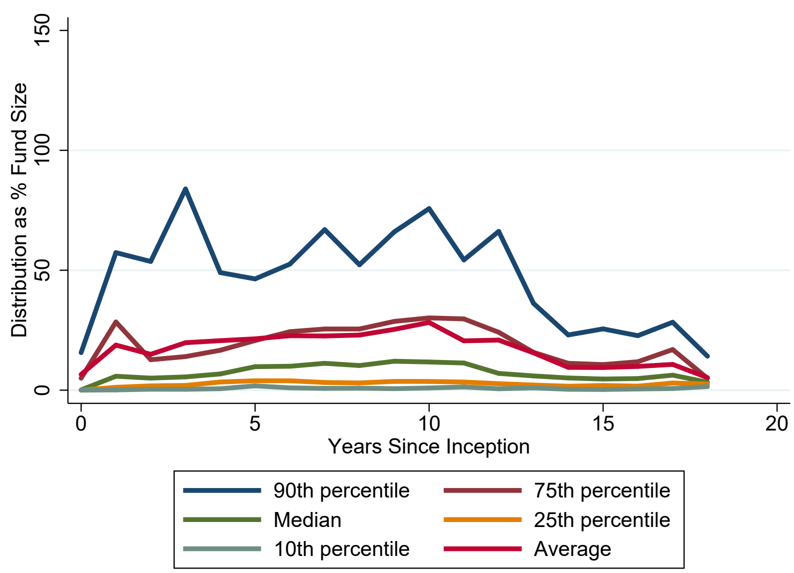


Figure 4: PME-GPME Wedge Using Market Risk SDF

We plot the PME - GPME wedge of artificially levered funds with different leverage k. We create artificially levered funds using impact funds, VC funds, or matched funds respectively, $C^{PE}_{i,t+h(j)}$ and the matched T-bill benchmark funds, $C^{Rf}_{if,t+h(j)}$

$$L^{PE}_{i,t+h(j)} = C^{PE}_{i,t+h(j)} + k(C^{PE}_{i,t+h(j)} - C^{Rf}_{if,t+h(j)})$$

We estimate the market risk SDF using replicating benchmarks for pooled impact and both benchmarks cash flows. We apply the same SDF on different levered cash flows to estimate GPME and use the log-utility CAPM SDF to estimate PME. The wedge is the difference between PME and GPME point estimates. The error bars are 95% confidence interval using bootstrap standard errors from 1,000 bootstrap samples of impact or VC funds.

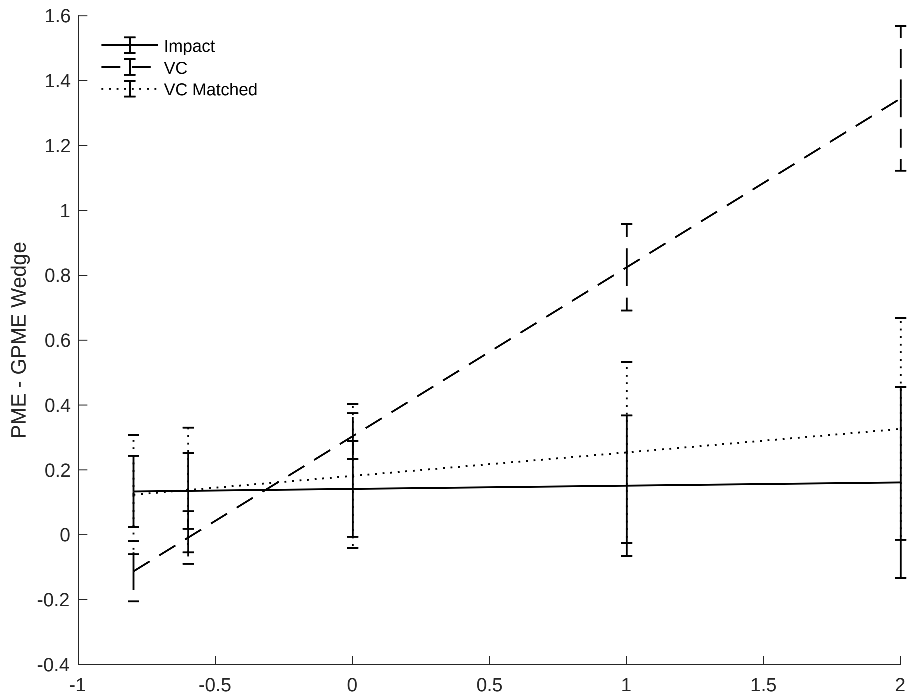


Figure 5: PME-GPME Wedge of Long-Short Portfolio

We create two portfolios, one that is long VC funds and short impact funds, and the other that is long matched funds and short impact funds. Both portfolios presented here are value-weighted. We create artificially levered cash flows of each strategy similar to Figure 4. For each level of artificial leverage k, we plot the difference between the PME and GPME point estimate of the long-short portfolio. The error bars are 95% confidence interval using bootstrap standard errors from 1,000 bootstrap samples of the long-short portfolio.

(a) Value-Weighted

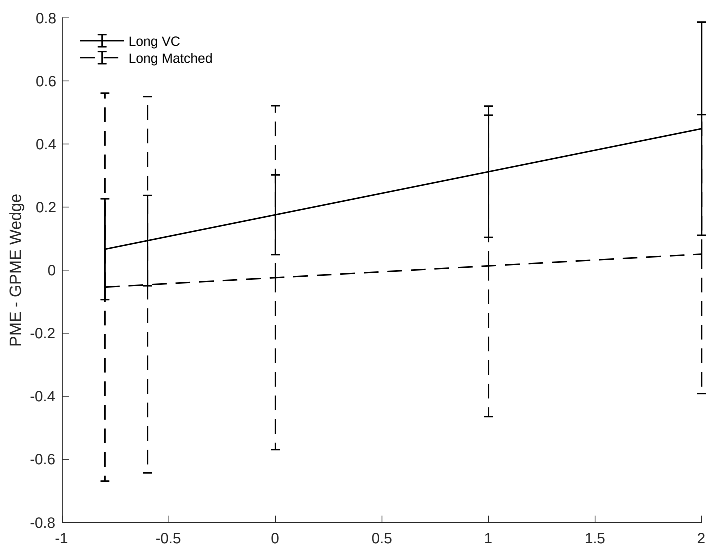


Figure 6: PME-GPME Wedge with Sustainability Index

We plot the PME - GPME wedge of artificially levered funds with different leverage k. We create artificially levered funds as in Figure 4. We estimate the sustainability index SDF using replicating benchmarks for pooled impact and both benchmarks cash flows. We apply the same SDF on different levered cash flows to estimate GPME and use the log-utility CAPM SDF to estimate PME. The wedge is the difference between PME and $GPME_{SI}$ point estimates. The error bars are 95% confidence interval using bootstrap standard errors from 1,000 bootstrap samples of impact or VC funds. Since the sustainability index has a relatively small equity premium relative to the log-utility benchmark (i.e., $\log E[R_{SI,t+1}] - r_f - \sigma_{SI}^2 < 0$), a negative relationship between the $PME - GPME_{SI}$ wedge and k indicates $\beta_{SI} > 0$, and a positive relationship between the $PME - GPME_{SI}$ wedge and k indicates $\beta_{SI} < 0$.

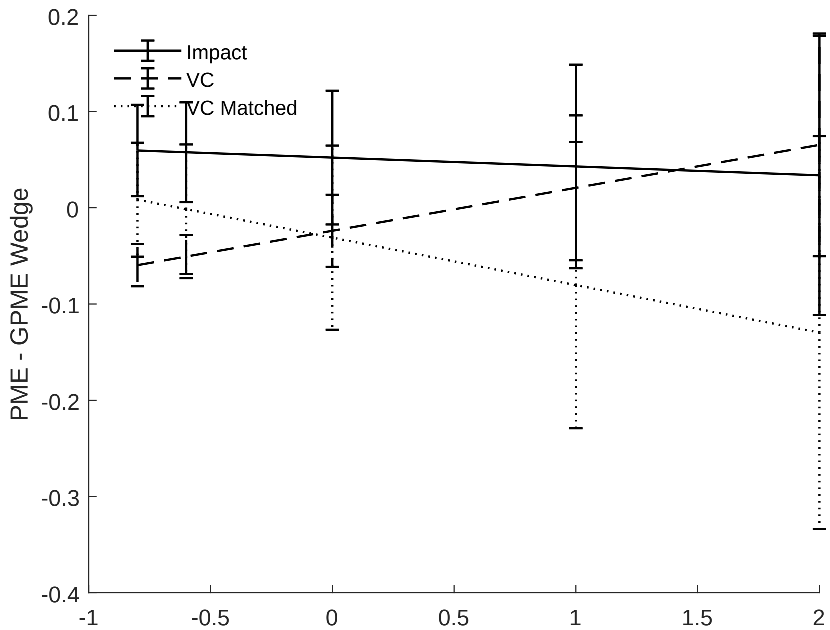


Table 1: Summary Statistics

Impact fund data comes from the IFD. VC and matched fund data come from Preqin. All samples cover vintages from 1997 through 2016 with transaction dates from 1999 to 2017. Impact fund statistics are reported in columns 1-3, VC fund statistics are in columns 4-6, and matched fund statistics are in columns 7-9. We report the mean and median for each sample of funds. The vintage year is the year of fund inception and fund size is the total committed capital raised by the fund, reported in millions USD. The PME is the ratio described in Equation 1, which we calculate using quarterly data when available and annual data for the remaining impact funds. VC and matched PMEs are all calculated using quarterly data. We also calculate the PME using only quarterly data and separately using only annual data. We do this to demonstrate that the two frequencies are very similar. We report other absolute measures of performance, the cash flow multiple and IRR in percent. Finally, we report characteristics about the cash flow profile of each set of funds. Effective years is the number of years funds in each sample are open. We also report the number of total cash flows per fund, as well as the number of contributions and distributions separately.

	Impact			VC			Matched Benchmark		
	N	Mean	Median	N	Mean	Median	N	Mean	Median
Vintage	51	2009.5	2010	483	2005.9	2006	51	2009.5	2010
Fund Size (Mill$)	51	95.3	74.4	483	382.2	280	51	154.4	125
PME	51	0.777	0.767	483	0.882	0.815	51	0.948	0.871
PME (Quarterly only)	44	0.769	0.751	483	0.879	0.817	51	0.951	0.877
PME (Annual only)	51	0.79	0.805	483	0.88	0.812	51	0.958	0.882
Multiple	51	1.123	1.081	483	1.331	1.183	51	1.351	1.186
IRR (%)	51	1.526	3.327	483	4.692	4.108	51	8.666	9.319
Effective Years	51	6.7	6.0	483	11.0	11	51	6.1	5.5
# Cash Flows per Fund	51	18.7	16	483	30.6	30	51	19.0	18
# Contributions	51	13.3	12	483	19.3	18	51	13.4	13
# Distributions	51	5.4	2	483	11.3	10	51	5.6	3

Table 2: Estimated SDF with Market Risk

The SDF for both PME and Market Risk GPME is given by $M^*_{t+1} = \exp(a - b_1 r^M_{t+1})$. The SDF corresponding to the PME is the SDF for a log-utility model, where $a = 0$ and $b_1 = 1$. The SDF corresponding to the Market Risk GPME relaxes this assumption. We estimate this SDF using benchmark portfolios to replicate pooled VC, matched, and impact fund cash flows, as described in Appendix A.

	PME SDF	Market Risk SDF
a	0	0.180
		(0.065)
b_1	1	3.765
		(0.738)

Table 3: Estimated Performance Relative to Market Risk

We compute PME and GPME relative to the market risk factor using the SDF parameters from Table 2. We report the standard error in parentheses below the estimate, and the p-value of the J-test of whether the estimate is zero in brackets on the third line. Panel (a) estimates performance for impact funds, Panel (b) for VC funds, and Panel (c) for matched funds. All panels use the same SDF parameters.

Panel (a): Impact funds

	PME	Market Risk GPME
Estimate	-0.155	-0.296
(sd)	(0.026)	(0.208)
[J-stat p-value]	[0.000]	[0.155]

Panel (b): VC funds

	PME	Market Risk GPME
Estimate	-0.132	-0.436
(sd)	(0.049)	(0.115)
[J-stat p-value]	[0.007]	[0.000]

Panel (c): Matched funds

	PME	Market Risk GPME
Estimate	-0.026	-0.207
(sd)	(0.011)	(0.068)
[J-stat p-value]	[0.445]	[0.323]

Table 4: Estimated GPME of Long-Short Portfolio

We compute PME and Market Risk GPME for two portfolios, one that is long VC funds and short impact funds, and the other that is long matched funds and short impact funds. We report the standard error in parentheses below the estimate, and the p-value of the J-test of whether the estimate is zero in brackets on the third line. Panel (a) estimates performance for the value-weighted VC-impact portfolio, Panel (b) for the equal weighted VC-impact portfolio, Panel (c) for the value-weighted matched-impact portfolio, and Panel (d) for the equal-weighted matched-impact portfolio. Both panels use the same SDF parameters from Table 2.

Panel (a): VC-impact value-weighted long-short portfolio

	PME	Market Risk GPME
Estimate	0.023	-0.152
(sd)	(0.043)	(0.080)
[J-stat p-value]	[0.582]	[0.056]

Panel (b): Matched-impact value-weighted long-short portfolio

	PME	Market Risk GPME
Estimate	0.083	0.080
(sd)	(0.032)	(0.147)
[J-stat p-value]	[0.009]	[0.586]

Table 5: Estimated SDF with Sustainability Index

The SDF for both PME and GPME relative to the Sustainability Index is given by $M^*_{t+1} = \exp(a - b_1 r^{SI}_{t+1})$. The SDF corresponding to the PME is the SDF for a log-utility model, where $a = 0$ and $b_1 = 1$. The SDF corresponding to the SI GPME relaxes this assumption. We estimate this SDF using benchmark portfolios to replicate pooled VC, matched, and impact fund cash flows, as described in Appendix A.

	SI PME SDF	SI SDF
a	0	0.045
		(0.036)
b_1	1	2.273
		(0.841)

Table 6: Estimated Performance Relative to Sustainability Index

We compute the PME and GPME relative to the Sustainability Index using the SDF parameters from Table 5. We report the standard error in parentheses below the estimate, and the p-value of the J-test of whether the estimate is zero in brackets on the third line. Panel (a) estimates performance for impact funds, Panel (b) for VC funds, and Panel (c) for matched funds. Both panels use the same SDF parameters.

Panel (a): Impact funds

	SI PME	SI GPME
Estimate	-0.048	-0.100
(sd)	(0.033)	(0.223)
[J-stat p-value]	[0.143]	[0.654]

Panel (b): VC funds

	SI PME	SI GPME
Estimate	0.112	0.136
(sd)	(0.075)	(0.095)
[J-stat p-value]	[0.137]	[0.150]

Panel (c): Matched funds

	SI PME	SI GPME
Estimate	0.185	0.216
(sd)	(0.015)	(0.075)
[J-stat p-value]	[0.000]	[0.352]

Table 7: GPME with Multi-Factor SDF

We compute the GPME for different factor models: one-factor model with market factor only, two-factor model with market and small growth, and two-factor model with market and sustainability index. We report the standard error in parentheses below the estimate, and the p-value of the J-test of whether the estimate is zero in brackets on the third line. Panel (a) estimates performance for impact funds, Panel (b) for VC funds, and Panel (c) for matched funds. Both panels use the same SDF parameters; each column has its own set of SDF parameters corresponding to the relevant model.

Panel (a): Impact funds

	Market Factor Only	Market and Small Growth	Market and Sustainability
Estimate	-0.296	-0.296	-0.315
(sd)	(0.208)	(0.186)	(0.384)
[J-stat p-value]	[0.155]	[0.112]	[0.412]

Panel (b): VC funds

	Market Factor Only	Market and Small Growth	Market and Sustainability
Estimate	-0.436	-0.413	-0.428
(sd)	(0.115)	(0.099)	(0.113)
[J-stat p-value]	[0.000]	[0.000]	[0.000]

Panel (c): Matched funds

	Market Factor Only	Market and Small Growth	Market and Sustainability
Estimate	-0.207	-0.198	-0.292
(sd)	(0.068)	(0.060)	(0.125)
[J-stat p-value]	[0.323]	[0.288]	[0.448]

A Estimation Method

A.1 Pricing Cash Flows for Public Market Replicating Portfolios

We follow Korteweg and Nagel (2016) in the construction and pricing of public market replicating cash flows. We pay into the replicating fund at the same time and with the same magnitude as PE fund contributions. When the PE fund makes a distribution at time $t + h(j)$, we assume that public market replicating funds make a payout equal to the sum of

1. Return accumulated since $t + h(j - 1)$
2. A fraction π_j of capital in the fund since $t + h(j - 1)$:

$$\pi_j = \min\left(\frac{h(j) - p}{10 - p}, 1\right)$$

Where p is time since last payout in years.

This second piece constrains the effective life of the replicating funds to 10 years. In robustness tests, we found little change in results when we extended the effective life of replicating funds beyond 10 years. The final period NAV is treated as a distribution in this set up. This opens our analysis up to potential issues related to the manipulation of NAV by fund managers, as discussed in Brown, Gredil, and Kaplan (2019). We also rely heavily on this single period distribution for very young funds, that have not had many distributions.

We thus have cash flows for PE funds, risk-free rate funds f, and market funds M for the set of both benchmark funds (N_B funds) and impact funds (N_{Imp} funds). As in Korteweg and Nagel (2016), we form the following matrix of cash flows with dimensions $(N_B + N_{Imp}) \times 3 \times J$:

$$Y_{i,t+h(j)} = \begin{pmatrix} C_{i,t+h(j)} \\ C_{if,t+h(j)} \\ C_{iM,t+h(j)} \end{pmatrix}$$

Pricing errors for each i of the $N_B + N_{Imp}$ funds are:

$$u_i(\theta) = \sum_{j=1}^{J} M_{t+h(j)}^{h(j)}(\theta) Y_{i,t+h(j)}$$

We form the GMM estimator as:

$$\hat{\theta} = \arg\min_{\theta} \left(\frac{1}{N} \sum_i u_i(\theta) \right)' \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\frac{1}{N} \sum_i u_i(\theta) \right)$$

Where we put positive weight only on the replicating funds to ensure that our SDF perfectly prices these cash flows. Importantly, the set of u_i is for all $N_B + N_{Imp}$ funds. We pool the cash flows together in order to have a consistent SDF for comparing benchmark and impact funds. Additionally, the GMM procedure requires both a large number of funds and non-overlapping time periods. Estimating the SDF from the pooled set of replicating funds requires an assumption that our replicating funds' cash flow profiles are relatively similar to the original set of benchmark and impact funds. This assumption is also present in Korteweg and Nagel (2016). On top of this assumption, we also need to assume that the timing of cash flows between VC, matched, and impact funds are similar. This assumption allows us to use the same SDF to price VC, matched, and impact fund cash flows, attributing differences in performance to group-level differences rather than to bias in the timing of cash flows. Because we have more VC replicating funds in our SDF estimation, a violation of this assumption will result in a GPME that reflects differences in the realization of the SDF for each set of cash flows more than it reflects differences in performance between VC, matched, and impact funds. Fortunately, Figures 2 and 3 provide evidence in favor of the similarity of cash flow profiles.

With the parameters of the SDF that correctly prices the benchmark cash flows, we discount cash flows of impact, VC, and matched funds separately. We use a J-statistic to test whether the pricing errors of each set of funds are jointly zero. Thus we test whether the GPME estimates for $i \in N_{VC}$ are jointly zero separately from whether the GPME estimates for $i \in N_{Imp}$ are jointly zero, and likewise for $i \in N_{Match}$.

A.2 Spectral Density Matrix Adjustments

There is potentially substantial correlation between the u_i from the previous section if they are measured over overlapping time periods. We follow Korteweg and Nagel (2016) and correct for this correlation using the following spectral density matrix in our tests:

$$\hat{S} = \hat{\Lambda}^{\frac{1}{2}} \hat{\Gamma} \hat{\Lambda}^{\frac{1}{2}}$$

Where the matrix of correlations is given by:

$$\hat{\Gamma} = \left[\frac{1}{N}\sum_i \text{diag}(u_i \circ u_i)\right]^{-\frac{1}{2}} \left(\frac{1}{N}\sum_i u_i u_i'\right) \left[\frac{1}{N}\sum_i \text{diag}(u_i \circ u_i)\right]^{-\frac{1}{2}}$$

And the diagonal matrix of variances is given by:

$$\hat{\Lambda} = \frac{1}{N}\sum_k \sum_i \max(1 - d(i,k)/\bar{d}, 0)\text{diag}(u_i \circ u_k)$$

$$\text{where } d(i,k) = 1 - \frac{\min[t(i)+h(i), t(k)+h(k)] - \max[t(i), t(k)]}{\max[t(i)+h(i), t(k)+h(k)] - \min[t(i), t(k)]}$$

Our analysis extends Korteweg and Nagel (2016) by estimating the SDF on a set of benchmark funds that are a superset of the funds that are ultimately used to construct group-level GPME estimates. Specifically, we estimate SDF using the pooled sample (long sample) and incorporate the SDF point estimate and standard error in predicting GPME estimates on the impact, matched, and VC fund sample respectively (limited sample). We therefore adjust the spectral density matrix to account for estimation error, following the methods developed in Stambaugh (1997) and Lynch and Wachter (2013):

$$\hat{S} = \begin{bmatrix} \hat{S}_{11} & \hat{S}_{11}\hat{B}_{21}^T \\ \hat{B}_{21}\hat{S}_{11} & \hat{\Sigma} + \hat{B}_{21}\hat{S}_{11}\hat{B}_{21}^T \end{bmatrix}$$

where $\hat{S}_{11}$ is the $(K+1) \times (K+1)$ spectral density matrix estimated from the pooled-sample pricing errors using the Korteweg and Nagel (2016) method, where K is the number of benchmark funds for the corresponding VC payoffs. $\hat{B}_{21}$ is the coefficients of a multivariate regression of the limited sample moments on the long sample moments and $\hat{\Sigma}$ is the residual matrix of the moment regressions.

The final spectral density matrix, $\hat{S}^{\mathcal{L}}$ also needs to account for the difference in sample length, so we further adjust the block entries as follows:

$$\hat{S}^{\mathcal{L}} = \begin{bmatrix} \lambda\hat{S}_{11} & \lambda\hat{S}_{11}\hat{B}_{21}^T \\ \lambda\hat{B}_{21}\hat{S}_{11} & \hat{\Sigma} + \hat{B}_{21}\hat{S}_{11}\hat{B}_{21}^T \end{bmatrix}$$

where λ is the ratio of the sample length of the limited and long sample,

$$\lambda = \frac{N_{short}}{N}$$

In our case, the limited-sample moments are exactly the same as the subset of corresponding long-sample moments, since the long sample is the super-set of funds in the limited sample. Therefore, we have

$$\hat{B}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting adjusted spectral density matrix $\hat{S}^{\mathcal{L}}$ is a $(2K+2) \times (2K+2)$ matrix, including the moments from VC payoff and benchmark payoffs from both the long and limited sample.

Note that, we do not need to use the efficient estimators developed by Lynch and Wachter (2013) as we are using a pre-specified weighting matrix instead of the optimal weighting matrix $W = S^{-1}$. Thus, the variance of moment conditions is

$$var(g) = (I - d(d'Wd)^{-1}d'W)\hat{S}(I - Wd(d'Wd)^{-1}d')$$

where W is a $(2K+2) \times (2K+2)$ zero matrix with diagonal entries of 1 for corresponding benchmark fund payoffs of the pooled sample, which are the only moments used to estimate the SDF. And a J-test on GPME is

$$g_{VC} var(g_{VC})^{-1} g_{VC} \sim \chi^2(1)$$

B Robustness

Table B.1: Estimated GPME of Equal-Weighted Long-Short Portfolio

We compute PME and Market Risk GPME for two portfolios, one that is long VC funds and short impact funds, and the other that is long matched funds and short impact funds. We report the standard error in parentheses below the estimate, and the p-value of the J-test of whether the estimate is zero in brackets on the third line. Panel (a) estimates performance for the equal weighted VC-impact portfolio, and Panel (b) for the equal-weighted matched-impact portfolio. Both panels use the same SDF parameters from Table 2.

Panel (a): VC-impact equal-weighted long-short portfolio

	PME	Market Risk GPME
Estimate	0.055	-0.132
(sd)	(0.035)	(0.074)
[J-stat p-value]	[0.123]	[0.076]

Panel (b): Matched-impact equal-weighted long-short portfolio

	PME	Market Risk GPME
Estimate	0.115	0.107
(sd)	(0.032)	(0.179)
[J-stat p-value]	[0.000]	[0.549]

Figure B.2: Equal-Weighted

We create two portfolios, one that is long VC funds and short impact funds, and the other that is long matched funds and short impact funds, both equal-weighted. We create artificially levered cash flows of this strategy. For each level of artificial leverage k, we plot the difference between the PME and GPME point estimate of this long-short portfolio. The error bars are 95% confidence interval using bootstrap standard errors from 1,000 bootstrap samples of the long-short portfolio.